Linear Functions

Formula : Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$, Midpoint $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$,

Distance $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, Slope of parallel lines $m_1 = m_2$

Slope of perpendicular lines $m_1 \bullet m_2 = -1$ (negative reciprocals)

1. Determine the slope, midpoint and the distance between the given points

a) (-3, 6) and (2, -7)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-7)}{-3 - 2} = \frac{6 + 7}{-5} = -\frac{13}{5}$$

$$M\left(\frac{x_x + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 2}{2}, \frac{6 + (-7)}{2}\right) = \left(-\frac{1}{2}, -\frac{1}{2}\right)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-3) - 2)^2 + (6 - (-7))^2} = \sqrt{(-5)^2 + (13)^2} = \sqrt{194}$$

b) (-4, -7) and (8, 4)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 4}{-4 - 8} = \frac{-11}{-12} = \frac{11}{12}$$

$$M\left(\frac{x_x + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4 + 8}{2}, \frac{-7 + 4}{2}\right) = \left(\frac{4}{2}, -\frac{3}{2}\right)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-4) - 8)^2 + (-7 - 4)^2} = \sqrt{(-12)^2 + (-11)^2} = \sqrt{265}$$

c) (4, -7) and (8, -12)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - (-12)}{4 - 8} = \frac{-7 + 12}{-4} = -\frac{5}{4}$$

$$M\left(\frac{x_x + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + 8}{2}, \frac{-7 + (-12)}{2}\right) = \left(\frac{12}{2}, -\frac{19}{2}\right)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 8)^2 + (-7 - (-12))^2} = \sqrt{(-4)^2 + (5)^2} = \sqrt{41}$$

d) (13, 17) and (-9, 5)

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{17-5}{13-(-9)}=\frac{12}{13+9}=\frac{12}{22}$$

$$M\left(\frac{x_x+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{13+(-9)}{2},\frac{17+5}{2}\right)=\left(\frac{4}{2},\frac{22}{2}\right)$$

$$d=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}=\sqrt{\left((13-(-9)\right)^2+\left(17-5\right)^2}=\sqrt{(22)^2+(12)^2}=\sqrt{628}$$

2. Determine the slope and y-intercept for each of the following:

a) 5x - 7y = 12

$$-7y=-5x+12$$
$$y=\frac{-5}{-7}x+\frac{12}{-7}$$
$$m=\frac{5}{7},b=-\frac{12}{7}$$

$$m=-\frac{A}{B}=-\frac{5}{-7}=\frac{5}{7}$$
$$b=\frac{C}{B}=\frac{12}{-7}=-\frac{12}{7}$$

b) –8x + 4y = -9

$$4y=8x-9$$
$$y=\frac{8}{4}x-\frac{9}{4}$$
$$m=\frac{8}{4},b=-\frac{9}{4}$$

$$m=-\frac{A}{B}=-\frac{-8}{4}=\frac{8}{4}$$
$$b=\frac{C}{B}=\frac{-9}{4}=-\frac{9}{4}$$

c) –2x + 5y = 11

$$5y=2x+11$$
$$y=\frac{2}{5}x+\frac{11}{5}$$
$$m=\frac{2}{5},b=\frac{11}{5}$$

$$m=-\frac{A}{B}=-\frac{-2}{5}=\frac{2}{5}$$
$$b=\frac{C}{B}=\frac{11}{5}$$

d) 4x – 7y = 13

$$-7y=-4x+13$$
$$y=\frac{-4}{-7}x+\frac{13}{-7}$$
$$m=\frac{4}{7},b=-\frac{13}{7}$$

$$m=-\frac{A}{B}=-\frac{-4}{-7}=-\frac{4}{7}$$
$$b=\frac{C}{B}=\frac{13}{-7}=-\frac{13}{7}$$

3. Determine the slope of the line parallel to and perpendicular to:

a) a line with slope of 2/3 $\quad // \Rightarrow m_2=\frac{2}{3}, \quad \perp \Rightarrow m_2=-\frac{3}{2}$

b) a line with slope of –5/7 $\quad // \Rightarrow m_2=-\frac{5}{7}, \quad \perp \Rightarrow m_2=\frac{7}{5}$

c) a line passing through (2, 6) and (-7, 2)

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{6-2}{2-(-7)}=\frac{4}{2+7}=\frac{4}{9}$$

$$// \Rightarrow m_2=\frac{4}{9}, \quad \perp \Rightarrow m_2=-\frac{9}{4}$$

d) a line with equation 7x – 3y = 12

$$-3y=-7x+12 \Rightarrow y=\frac{-7}{-3}x+\frac{12}{-3} \Rightarrow m=\frac{7}{3}$$

$$// \Rightarrow m_2=\frac{7}{3}, \quad \perp \Rightarrow m_2=-\frac{3}{7}$$

4. Determine the equation of the line given :

a) m = -3, b = -4

$$y = mx + b$$
$$y = -3x - 4$$
$$3x + y = -4$$

b) m = -4/7, b = 3/5

$$y = mx + b$$
$$y = -\frac{4}{7}x + \frac{3}{5} \Rightarrow 35y = -20x + 21$$
$$20x + 35y = 21$$

c) m = 3/4, (0, 5)

$$b = 5$$
$$y = mx + b$$
$$y = \frac{3}{4}x + 5 \Rightarrow 4y = 3x + 20$$
$$-3x + 4y = 20$$

d) m = -2/5, (4, -5)

$$(y_2 - y_2) = m(x_2 - x_1)$$
$$(y - (-5)) = -\frac{2}{5}(x - 4)$$
$$(y + 5) = -\frac{2}{5}(x + 3)$$
$$5y + 25 = -2x - 6 \Rightarrow 5y = -2x - 31$$
$$2x + 5y = -31$$

f) m = -3/5, b = 7

$$y = mx + b$$
$$y = -\frac{3}{5}x + 7 \Rightarrow 5y = -3x + 35$$
$$3x + 5y = 35$$

g) m = -3, (0, -7)

$$b = -7$$
$$y = mx + b$$
$$y = -3x - 7$$
$$3x + y = -7$$

h) m = -3, (-3, 5)

$$(y_2 - y_2) = m(x_2 - x_1)$$
$$(y - 5) = -3(x - (-3))$$
$$(y - 5) = -3(x + 3)$$
$$y - 5 = -3x - 9 \Rightarrow y = -3x - 4$$
$$3x + y = -4$$

i) (-4, -7), (-2, 6)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 6}{-4 - (-2)} = \frac{-13}{-4 + 2} = \frac{13}{2}$$
$$(y_2 - y_2) = m(x_2 - x_1)$$
$$(y - (-7)) = \frac{13}{2}(x - (-4))$$
$$(y + 7) = \frac{13}{2}(x + 4)$$
$$2y + 14 = 13x + 52 \Rightarrow 2y = 13x + 38$$
$$-13x + 2y = 38$$

e) (-6, 4), (7, -2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{-6-7} = \frac{4+2}{-13} = -\frac{6}{13}$$

$$(y_2 - y_2) = m(x_2 - x_1)$$

$$(y-4) = -\frac{6}{13}(x-(-6))$$

$$(y-4) = -\frac{6}{13}(x+6)$$

$$13y - 52 = -6x - 36 \Rightarrow 13y = -6x + 16$$

$$6x + 13y = 16$$

j) through (-6, 3) and parallel to the equation 6x – 5y = 2

$$m \Rightarrow -5y = -6x + 2 \Rightarrow y = \frac{-6}{-5}x + \frac{2}{-5} \Rightarrow m_1 = \frac{6}{5} \therefore m_2 = \frac{6}{5}$$

$$\text{point} \Rightarrow (-6, 3)$$

$$(y_2 - y_2) = m(x_2 - x_1)$$

$$(y-3) = \frac{6}{5}(x-(-6))$$

$$(y-3) = \frac{6}{5}(x+6)$$

$$5y - 15 = 6x + 36 \Rightarrow 5y = 6x + 51$$

$$-6x + 5y = 51$$

k) through (-2, -5) and perpendicular to the equation –4x + 3y = 2

$$m \Rightarrow 3y = 4x + 2 \Rightarrow y = \frac{4}{3}x + \frac{2}{3} \Rightarrow m_1 = \frac{4}{3} \therefore m_2 = -\frac{3}{4}$$

$$\text{point} \Rightarrow (-2, -5)$$

$$(y_2 - y_2) = m(x_2 - x_1)$$

$$(y-(-5)) = -\frac{3}{4}(x-(-2))$$

$$(y+5) = -\frac{3}{4}(x+2)$$

$$4y + 20 = -3x - 6 \Rightarrow 4y = -3x - 26$$

$$3x + 4y = -26$$

l) through (-3, 3) and parallel to the line defined by points (2, 6) and (-5, 11)

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{6-11}{2-(-5)}=\frac{-5}{2+5}=\frac{-5}{7}\Rightarrow m_1=-\frac{5}{7}\therefore m_2=-\frac{5}{7}$$

$$\text{point }\Rightarrow(-3,3)$$

$$(y_2-y_2)=m(x_2-x_1)$$

$$(y-3)=-\frac{5}{7}(x-(-3))$$

$$(y-3)=-\frac{5}{7}(x+3)$$

$$7y-21=-5x-15\Rightarrow 7y=-5x+6$$

$$5x+7y=6$$

m) through (-7, 2) and perpendicular to the line defined by the points (7, 4) and (-3, 6)

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-6}{7-(-3)}=\frac{-2}{7+3}=\frac{-2}{10}\Rightarrow m_1=-\frac{2}{10}\therefore m_2=\frac{10}{2}=\frac{5}{1}=5$$

$$\text{point }\Rightarrow(-7,2)$$

$$(y_2-y_2)=m(x_2-x_1)$$

$$(y-2)=5(x-(-7))$$

$$(y-2)=5(x+7)$$

$$y-2=5x+35\Rightarrow y=5x+37$$

$$-5x+y=37$$

n) perpendicular bisector of the line segment defined by the points (-7, 4) and (3, -8)

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-(-8)}{-7-3}=\frac{4+8}{-10}=\frac{12}{-10}\Rightarrow m_1=-\frac{12}{10}=-\frac{6}{5}\therefore m_2=\frac{5}{6}$$

$$M\left(\frac{x_x+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-7+3}{2},\frac{4+(-8)}{2}\right)=\left(-\frac{4}{2},-\frac{4}{2}\right)=(-2,-2)$$

$$(y_2-y_2)=m(x_2-x_1)$$

$$(y-(-2))=\frac{5}{6}(x-(-2))$$

$$(y+2)=\frac{5}{6}(x+2)$$

$$6y+12=5x+10\Rightarrow 6y=5x-2$$

$$-5x+6y=-2$$